7.4 Evaluate Logarithms and Graph Logarithmic Functions

Before You evaluated and graphed exponential functions.

Now You will evaluate logarithms and graph logarithmic functions.

Why? So you can model the wind speed of a tornado, as in Example 4.

Key Vocabulary
- logarithm of y with base b
- common logarithm
- natural logarithm

You know that $2^2 = 4$ and $2^3 = 8$. However, for what value of x does $2^x = 6$? Mathematicians define this x-value using a *logarithm* and write $x = \log_2 6$. The definition of a logarithm can be generalized as follows.

KEY CONCEPT *For Your Notebook*

Definition of Logarithm with Base b

Let b and y be positive numbers with $b \neq 1$. The **logarithm of y with base b** is denoted by $\log_b y$ and is defined as follows:

$$\log_b y = x \quad \text{if and only if} \quad b^x = y$$

The expression $\log_b y$ is read as "log base b of y."

This definition tells you that the equations $\log_b y = x$ and $b^x = y$ are equivalent. The first is in *logarithmic form* and the second is in *exponential form*.

EXAMPLE 1 Rewrite logarithmic equations

	Logarithmic Form	Exponential Form
a.	$\log_2 8 = 3$	$2^3 = 8$
b.	$\log_4 1 = 0$	$4^0 = 1$
c.	$\log_{12} 12 = 1$	$12^1 = 12$
d.	$\log_{1/4} 4 = -1$	$\left(\frac{1}{4}\right)^{-1} = 4$

Parts (b) and (c) of Example 1 illustrate two special logarithm values that you should learn to recognize. Let b be a positive real number such that $b \neq 1$.

Logarithm of 1	Logarithm of b with Base b
$\log_b 1 = 0$ because $b^0 = 1$.	$\log_b b = 1$ because $b^1 = b$.

✓ GUIDED PRACTICE for Example 1

Rewrite the equation in exponential form.

1. $\log_3 81 = 4$ **$3^4 = 81$** **2.** $\log_7 7 = 1$ **$7^1 = 7$** **3.** $\log_{14} 1 = 0$ **$14^0 = 1$** **4.** $\log_{1/2} 32 = -5$ **$\left(\frac{1}{2}\right)^{-5} = 32$**

1 PLAN AND PREPARE

Warm-Up Exercises

Transparency Available

1. Find the inverse of the function $y = 3x - 5$. **$y = \frac{1}{3}x + \frac{5}{3}$**

2. An account that pays 3% annual interest compounded continuously has a balance of $10,000 on June 1, 2008. If no money is added, what is the balance on June 1, 2010? **about $10,618.37**

Notetaking Guide

Transparency Available

Promotes interactive learning and notetaking skills, pp. 195–198.

Pacing

Basic: 2 days

Average: 2 days

Advanced: 2 days

Block: 0.5 block with 7.3
0.5 block with 7.5

- See Teaching Guide/Lesson Plan.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1, p. 477

What is the relationship between exponential and logarithmic functions? **Tell students they will learn how to answer this question by graphing logarithmic functions and comparing their graphs to those of exponential functions.**

NCTM STANDARDS

Standard 2: Understand functions

Standard 10: Use representations to solve problems

Resource Planning Guide

Chapter Resource Book
- Teaching Guide/Lesson Plan (pp. 38–39)
- Activity Master (p. 40)
- Practice levels A, B, C (pp. 42–44)
- Study Guide (pp. 45–46)
- Catch-up for Absent Students (p. 47)
- Problem Solving Workshop (p. 48)
- Challenge (p. 50)

Workbooks
- Notetaking Guide (pp. 195–198)
- Practice Workbook (pp. 114–115)

Teaching Options
- **Power Presentations CD-ROM** provides dynamic electronic teaching resources for the classroom.
- **Activity Generator CD-ROM** provides editable activities for all ability levels.

Interactive Technology
- Easy Planner
- Power Presentations CD-ROM
- Activity Generator CD-ROM
- Animated Algebra
- Test Generator CD-ROM
- Online Quiz
- eWorkbook
- eEdition
- @HomeTutor

Resources for English Learners
- Quick Reference for English Learners
- Spanish Study Guide
- Multi-Language Visual Glossary
- Student Resources in Spanish

See also the Algebra 2 Toolkit for more strategies for meeting individual needs.

Motivating the Lesson

Ask students who have studied chemistry to explain the meaning and importance of pH value. Explain that the pH formula uses common logarithms, which students will study in this lesson.

3 TEACH

Extra Example 1

Rewrite the equation in exponential form.

a. $\log_2 32 = 5$ $2^5 = 32$

b. $\log_{10} 1 = 0$ $10^0 = 1$

c. $\log_9 9 = 1$ $9^1 = 9$

d. $\log_{1/5} 25 = -2$ $\left(\frac{1}{5}\right)^{-2} = 25$

Extra Example 2

Evaluate the logarithm.

a. $\log_3 81$ 4

b. $\log_{1/4} 256$ -4

c. $\log_{10} 0.001$ -3

d. $\log_{64} 2$ $\frac{1}{6}$

Extra Example 3

Use a calculator to evaluate the logarithm.

a. $\log 0.85$ -0.071

b. $\ln 22$ 3.091

Extra Example 4

The sales of a certain video game can be modeled by $y = 20 \ln (x - 1) + 35$, where y is the monthly number (in thousands) of games sold during the xth month after the game is released for sale ($x > 1$). Estimate the number of video games sold during the 10th month after the game is released. **about 79,000 games**

EXAMPLE 2 Evaluate logarithms

Evaluate the logarithm.

a. $\log_4 64$ **b.** $\log_5 0.2$ **c.** $\log_{1/5} 125$ **d.** $\log_{36} 6$

Solution

To help you find the value of $\log_b y$, ask yourself what power of b gives you y.

a. 4 to what power gives 64? $4^3 = 64$, so $\log_4 64 = 3$.

b. 5 to what power gives 0.2? $5^{-1} = 0.2$, so $\log_5 0.2 = -1$.

c. $\frac{1}{5}$ to what power gives 125? $\left(\frac{1}{5}\right)^{-3} = 125$, so $\log_{1/5} 125 = -3$.

d. 36 to what power gives 6? $36^{1/2} = 6$, so $\log_{36} 6 = \frac{1}{2}$.

SPECIAL LOGARITHMS A **common logarithm** is a logarithm with base 10. It is denoted by $\log_{10}$ or simply by log. A **natural logarithm** is a logarithm with base e. It can be denoted by $\log_e$, but is more often denoted by ln.

Common Logarithm	Natural Logarithm
$\log_{10} x = \log x$	$\log_e x = \ln x$

Most calculators have keys for evaluating common and natural logarithms.

EXAMPLE 3 Evaluate common and natural logarithms

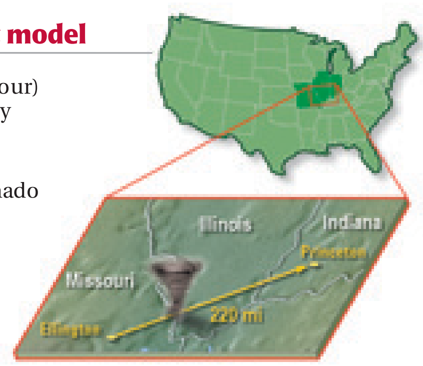
Expression	Keystrokes	Display	Check
a. log 8	LOG 8) ENTER	0.903089987	$10^{0.903} \approx 8$ ✓
b. ln 0.3	LN .3) ENTER	−1.203972804	$e^{-1.204} \approx 0.3$ ✓

EXAMPLE 4 Evaluate a logarithmic model

TORNADOES The wind speed s (in miles per hour) near the center of a tornado can be modeled by

$$s = 93 \log d + 65$$

where d is the distance (in miles) that the tornado travels. In 1925, a tornado traveled 220 miles through three states. Estimate the wind speed near the tornado's center.



Not drawn to scale

Solution

$s = 93 \log d + 65$ **Write function.**

$= 93 \log 220 + 65$ **Substitute 220 for d.**

$\approx 93(2.342) + 65$ **Use a calculator.**

$= 282.806$ **Simplify.**

▶ The wind speed near the tornado's center was about 283 miles per hour.

Differentiated Instruction

Below Level The concept of logarithms may cause confusion. After discussing **Example 1**, give students practice in rewriting a given exponential equation in logarithmic form. Prepare a diagram to show students how the three numbers involved move when you change from logarithmic to exponential form, and vice versa. Stress that the base is the same in both forms, just written in a different position, and that the logarithm is the exponent. Students may also benefit from a worksheet of mixed practice in changing between the two forms.

See also the Algebra 2 Toolkit for more strategies.

GUIDED PRACTICE for Examples 2, 3, and 4

Evaluate the logarithm. Use a calculator if necessary.

5. $\log_2 32$ **5** **6.** $\log_{27} 3$ $\frac{1}{3}$ **7.** $\log 12$ **about 1.079** **8.** $\ln 0.75$ **about −0.288**

9. WHAT IF? Use the function in Example 4 to estimate the wind speed near a tornado's center if its path is 150 miles long. **about 267 mi/h**

INVERSE FUNCTIONS By the definition of a logarithm, it follows that the logarithmic function $g(x) = \log_b x$ is the inverse of the exponential function $f(x) = b^x$. This means that:

$$g(f(x)) = \log_b b^x = x \quad \text{and} \quad f(g(x)) = b^{\log_b x} = x$$

EXAMPLE 5 Use inverse properties

Simplify the expression.

a. $10^{\log 4}$ **b.** $\log_5 25^x$

Solution

a. $10^{\log 4} = 4$ $b^{\log_b x} = x$

b. $\log_5 25^x = \log_5 (5^2)^x$ Express 25 as a power with base 5.

$= \log_5 5^{2x}$ Power of a power property

$= 2x$ $\log_b b^x = x$

EXAMPLE 6 Find inverse functions

Find the inverse of the function.

a. $y = 6^x$ **b.** $y = \ln (x + 3)$

REVIEW INVERSES
For help with finding inverses of functions, see p. 437.

Solution

a. From the definition of logarithm, the inverse of $y = 6^x$ is $y = \log_6 x$.

b. $y = \ln (x + 3)$ Write original function.

$x = \ln (y + 3)$ Switch x and y.

$e^x = y + 3$ Write in exponential form.

$e^x - 3 = y$ Solve for y.

▶ The inverse of $y = \ln (x + 3)$ is $y = e^x - 3$.

GUIDED PRACTICE for Examples 5 and 6

Simplify the expression.

10. $8^{\log_8 x}$ **x** **11.** $\log_7 7^{-3x}$ **−3x** **12.** $\log_2 64^x$ **6x** **13.** $e^{\ln 20}$ **20**

14. Find the inverse of $y = 4^x$. $y = \log_4 x$

15. Find the inverse of $y = \ln (x - 5)$. $y = e^x + 5$

Extra Example 5

Simplify the expression.

a. $e^{\ln 9}$ **9**

b. $\log_3 27^x$ **3x**

Key Questions to Ask for Example 5

- In Example 5a, what is the base of the exponential form? **10; log 4 means $\log_{10} 4$.**
- In Example 5b, why is 25 rewritten as 5^2? **To match the base of 25^x to the logarithmic base.**

Extra Example 6

Find the inverse of the function.

a. $y = 8^x$ $y = \log_8 x$

b. $y = \ln (x - 4)$ $y = e^x + 4$

Key Question to Ask for Example 6

- For an exponential function and a logarithmic function to be inverses, what must the two functions have in common? **They must have the same base.**

Mathematical Reasoning

Both $y = b^x$ and $y = \log_b x$, where $b > 0$, $b \neq 1$ have inverses. What must be true of their graphs? **Their graphs pass the horizontal line test.**